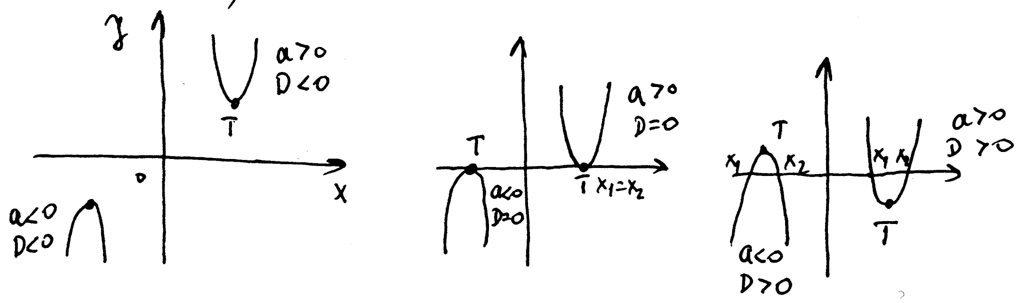
KVADRATNA FUNKCIJA

$f: \mathbb{R} \to \mathbb{R}$

$f(x) = ax^2 + bx + c,\ a \neq 0,\ a, b, c \in \mathbb{R}$

$f(x) = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$ kanonski oblik

$T\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$



Zadaci:

1. Za koju vrijednost parametra m je funkcija
   a) $y = x^2 + 2(m+7)x + m+5$ negativna za $\forall x \in \mathbb{R}$
   b) $y = (m+1)x^2 - 2(m+3)x + m - 1$ pozitivna $\forall x \in \mathbb{R}$
2. Data je funkcija $f(x) = (m-2)x^2 - 2mx + 2m - 3$
   a) za koju vrijednost parametra m je $f(x) < 0$ $\forall x \in \mathbb{R}$?
   b) odredi m tako da zbir recipročnih vrijednosti kvadrata korijena jednačine $f(x) = 0$ bude 2.
3. Za koje vrijednosti realnog parametra m je nejednakost
   $$\left| \frac{x^2 - mx + 1}{x^2 + x + 1} \right| < 3$$ ispunjena $\forall x \in \mathbb{R}$?
4. Za koju vrijednost parametra $m \in \mathbb{R}$ jednačina $(m-2)x^2 - (m+1)x + (m+1) = 0$ $(m \neq 2)$ ima rješenja suprotnih znakova?
5. Za koju vrijednost parametra m su korijeni jednačine $(m-2)x^2 - 2mx + m - 3 = 0$ $(m \neq 2)$ pozitivni?

I Data je kvadratna jednačina $(x-m)^2 = x+1$. Odredi realan broj m za svaki od sljedećih uslova: a) rješenja jednačine su suprotni brojevi
b) jedno rješenje jednačine jednako je nuli
c) rješenja jednačine su realni brojevi.

II Za koje vrijednosti realnog parametra k je trinom $(k-2)x^2 - 2kx + 3(k-3)$, $k \neq 2$
a) pozitivan $\forall x \in \mathbb{R}$ b) za koju vrijednost k vrijedi $x_1 - x_2 = 1$

III Data je kvadratna jednačina po x : $2x^2 + 2x + \cos\alpha = 0$ $(0 < \alpha < \pi)$
a) odrediti α ako je $\frac{1}{x_1} + \frac{1}{x_2} = \frac{4}{\sqrt{3}}$
b) pokazati da je tada $x_1^2 + x_2^2 < 1{,}9$, gdje su x_1 i x_2 rješenja date jednačine

Rješenja:

(1) a) i b) uradite sami

a) R: $m < -\frac{49}{9}$ tj. $m \in \left(-\infty, -\frac{49}{9}\right)$

b) nema rješenja

(2) a) $f(x) < 0 \iff a < 0 \land D < 0$

$m-2 < 0 \quad \land \quad D = 4m^2 - 4(m-2)(2m-3) < 0$

$m < 2 \quad \land \quad D = 4m^2 - 8m^2 + 28m - 24 < 0$

$-4m^2 + 28m - 24 < 0 \;/ : (-4)$

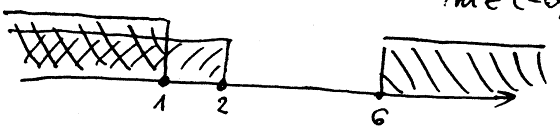
$m^2 - 7m + 6 > 0$

$m^2 - 6m - m + 6 > 0$

$m(m-6) - (m-6) > 0$

$m < 2 \quad \land \quad (m-1)(m-6) > 0$

$m \in (-\infty, 1) \cup (6, +\infty)$



Rješenje: $m \in (-\infty, 1)$

b) $x_1 + x_2 = \frac{2m}{m-2}$

$x_1 \cdot x_2 = \frac{2m-3}{m-2}$

$\frac{1}{x_1^2} + \frac{1}{x_2^2} = 2$

$\frac{x_2^2 + x_1^2}{(x_1 x_2)^2} = 2$

$\frac{x_1^2 + x_2^2}{(x_1 \cdot x_2)^2} = 2$

$\frac{(x_1+x_2)^2 - 2x_1 \cdot x_2}{(x_1 \cdot x_2)^2} = 2$

$$\frac{\frac{4m^2 - 2(2m-3)(m-2)}{\cancel{(m-2)^2}}}{\frac{2m-3}{\cancel{(m-2)^2}}} = 2$$

$4m^2 - 4m^2 + 14m - 12 = 2(4m^2 - 12m + 9)$

$4m^2 - 19m + 15 = 0$

$m_1 = \frac{15}{4}$

$m_2 = 1$

(3) $-3 < \frac{x^2 - mx + 1}{x^2 + x + 1} < 3$

$-3 < \frac{x^2 - mx + 1}{x^2 + x + 1} \quad \land \quad \frac{x^2 - mx + 1}{x^2 + x + 1} < 3$

$\frac{x^2 - mx + 1}{x^2 + x + 1} + 3 > 0 \quad \land \quad \frac{x^2 - mx + 1}{x^2 + x + 1} - 3 < 0$

$\frac{4x^2 - (m-3)x + 4}{x^2 + x + 1} > 0 \quad \land \quad \frac{-2x^2 - (m+3)x - 2}{x^2 + x + 1} < 0$

Kvadratni trinom $x^2 + x + 1 > 0 \;\; \forall x \in R$, pa pomnožimo samo brojnike

$4x^2-(m-3)x+4>0 \quad \wedge \quad -2x^2-(m+3)x-2<0$

$4x^2-(m-3)x+4>0 \quad \wedge \quad 2x^2+(m+3)x+2>0$

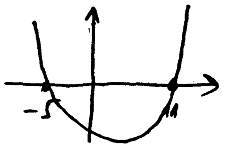
$f(x)>0 \iff a>0 \wedge D<0$

$D=(m-3)^2-4\cdot4\cdot4<0$ $\quad \wedge \quad$ $D=(m+3)^2-4\cdot2\cdot2<0$

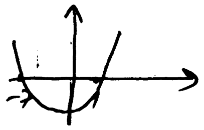
$m^2-6m+9-64<0$ $\qquad$ $m^2+6m+9-16<0$

$m^2-6m-55<0$ $\qquad$ $m^2+6m-7<0$

$(m+5)(m-11)<0$ $\qquad$ $(m-1)(m+7)<0$

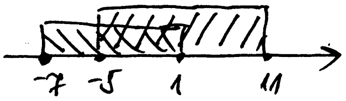


$m\in(-5,11)$



$m\in(-7,1)$

Rješenje:



$m\in(-5,1)$

4) $x_1\cdot x_2<0$

$x_1\cdot x_2=\frac{m+1}{m-2}<0$

	$-\infty$	-1	2	$+\infty$
$m+1$	$-$ 0	$+$	$+$	
$m-2$	$-$	$-$ 0	$+$	
$\frac{m+1}{m-2}$	$+$	$-$	$+$	

Rj: $m\in(-1,2)$

5) $D\geq0 \wedge x_1+x_2>0 \wedge x_1\cdot x_2>0$

$D=4m^2-4(m-2)(m-3)$

$D=4m^2-4(m^2-5m+6)$

$D=\cancel{4m^2}-\cancel{4m^2}+20m-24$

$20m-24\geq0$

$20m\geq24$

$m\geq\frac{6}{5}$

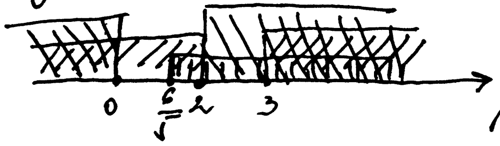
$x_1+x_2=\frac{2m}{m-2}>0$ $\qquad$ $x_1\cdot x_2=\frac{m-3}{m-2}>0$

	$-\infty$	0	2	$+\infty$
$2m$	$-$ 0	$+$	$+$	
$m-2$	$-$	$-$ 0	$+$	
$\frac{2m}{m-2}$	$+$	$-$	$+$	

	$-\infty$	2	3	$+\infty$
$m-3$	$-$	$-$ 0	$+$	
$m-2$	$-$ 0	$+$	$+$	
$\frac{m-3}{m-2}$	$+$	$-$	$+$	

$m\in(-\infty,0)\cup(2,+\infty)$ $\qquad$ $m\in(-\infty,2)\cup(3,+\infty)$

Rješ. zad:



$m\in(3,+\infty)$

LOGARITAMSKE I EKSPONENCIJALNE JEDNAČINE

1. Riješiti eksponencijalnu jednačinu $15\cdot 2^{x+1}+15\cdot 2^{2-x}=135$

Rj.

$$15\cdot 2^x\cdot 2+15\cdot 2^2\cdot 2^{-x}=135 \quad /:15$$
$$2^x\cdot 2+2^{-x}\cdot 4=9 \quad /\cdot 2^x$$
$$2\cdot 2^{2x}+4-9\cdot 2^x=0$$
$$2\cdot 2^{2x}-9\cdot 2^x+4=0$$
$$2^x=t$$
$$2\cdot t^2-9t+4=0$$
$$t_1=\frac{1}{2} \qquad t_2=4$$

$$2^x=\frac{1}{2} \qquad 2^x=4$$
$$2^x=2^{-1} \qquad 2^x=2^2$$
$$x=-1 \qquad x=2$$

2. Riješiti eksponencijalnu jednačinu $2^{|x+2|}-|2^{x+1}-1|=2^{x+1}+1$

Rj. $|x+2|=\begin{cases} x+2, & x+2\geq 0,\ x\geq -2 \\ -(x+2), & x<-2 \end{cases}$ $\quad |2^{x+1}-1|=\begin{cases} 2^{x+1}-1, & 2^{x+1}-1\geq 0,\ 2^{x+1}\geq 2^0,\ x\geq -1 \\ 1-2^{x+1}, & x<-1 \end{cases}$

	$-\infty$ … -2	-2 … -1	-1 … $+\infty$
$x+2$	$-$ 0	$+$	$+$
$2^{x+1}-1$	$-$	$-$ 0	$+$

1° $x\in(-\infty,-2)$

$$2^{-x-2}-(1-2^{x+1})=2^{x+1}+1$$
$$2^{-x-2}-1+\cancel{2^{x+1}}=\cancel{2^{x+1}}+1$$
$$2^{-x-2}=2$$
$$-x-2=1$$
$$-x=3$$
$$x=-3\in(-\infty,-2)$$

2° $x\in[-2,-1)$

$$2^{x+2}-(1-2^{x+1})=2^{x+1}+1$$
$$2^{x+2}-1+\cancel{2^{x+1}}=\cancel{2^{x+1}}+1$$
$$2^{x+2}=2$$
$$x+2=1$$
$$x=-1\in[-2,-1)$$

3° $x\in[-1,+\infty)$

$$2^{x+2}-2^{x+1}+1=2^{x+1}+1$$
$$2^{x+2}-2\cdot 2^{x+1}=0$$
$$2^{x+2}-2^{x+2}=0$$
$$\forall x\in R,\ x\geq -1$$

Kon. rješenje: $x=-3$ i $\forall x\in R\ (x\geq -1)$

3. Riješiti jednačinu: $\left(\frac{4}{9}\right)^{x} \cdot \left(\frac{27}{8}\right)^{x-1} = \frac{\log 4}{\log 8}$

$$\left(\left(\frac{2}{3}\right)^{2}\right)^{x} \cdot \left(\left(\frac{3}{2}\right)^{3}\right)^{x-1} = \frac{\log 4}{\log 8}$$

$$\left(\frac{2}{3}\right)^{2x} \cdot \left(\frac{3}{2}\right)^{3x-3} = \frac{\log 4}{\log 8}$$

$$\left(\frac{2}{3}\right)^{2x} \cdot \left(\frac{2}{3}\right)^{3-3x} = \frac{\log 2^{2}}{\log 2^{3}}$$

$$\left(\frac{2}{3}\right)^{2x} \cdot \left(\frac{2}{3}\right)^{3-3x} = \frac{2\cancel{\log 2}}{3\cancel{\log 2}}$$

$$\left(\frac{2}{3}\right)^{2x+3-3x} = \frac{2}{3}$$

$$\left(\frac{2}{3}\right)^{-x+3} = \frac{2}{3}$$

$$-x+3 = 1$$

$$-x = -2$$

$$x = 2$$

4. Riješiti jednačinu: $5^{\log x}-3^{\log x-1}=3^{\log x+1}-5^{\log x-1}$

Rj. $5^{\log x}+5^{\log x-1}=3^{\log x+1}+3^{\log x-1}$ $\quad x>0$

$5^{\log x}\left(1+\frac{1}{5}\right)=3^{\log x}\left(3+\frac{1}{3}\right)$

$5^{\log x}\frac{6}{5}=3^{\log x}\frac{10}{3}$

$5^{\log x}=3^{\log x}\cdot\frac{10^5}{3}\cdot\frac{5}{6_3}$

$\frac{5^{\log x}}{3^{\log x}}=\frac{5^2}{3^2}$

$\left(\frac{5}{3}\right)^{\log x}=\left(\frac{5}{3}\right)^2$

$\log x=2$

$\log x=\log 10^2$

Rj. $x=100$

5. $\log_x 3-\log_x 2=\frac{1}{2}$

$x>0,\ x\neq 1$

$\log_x \frac{3}{2}=\frac{1}{2}$

$\log_x \frac{3}{2}=\log_x x^{\frac{1}{2}}$

$\frac{3}{2}=x^{\frac{1}{2}}$

$\sqrt{x}=\frac{3}{2}$

Rj. $x=\frac{9}{4}>0$

6. Riješiti log. jednačinu $\log_2\sqrt{(1-x)^2}=3$

$\log_2|1-x|=3$

$x-1\neq 0 \quad x\neq 1$

$|1-x|=\begin{cases}1-x, & -x>-1,\ x<1\\ x-1, & x>1\end{cases}$

1° $x\in(-\infty,1)$

$\log_2(1-x)=3$

$\log_2(1-x)=\log_2 2^3$

$1-x=8$

$-x=7$

$x=-7\in(-\infty,1)$

2° $x\in(1,+\infty)$

$\log_2(x-1)=3$

$\log_2(x-1)=\log_2 2^3$

$x-1=8$

$x=9\in(1,+\infty)$

7. Riješi log. jednadžbu $\log x^2 + \log^2|x| = 3$

Rj. $2\log|x| + \log^2|x| = 3$ Definiciono područje: $x \neq 0$

$\log^2|x| + 2\log|x| - 3 = 0$

$\log|x| = t$

$t^2 + 2t - 3 = 0$

$t_1 = -3 \quad , \quad t_2 = 1$

$\log|x| = 1$

$\log|x| = \log 10$

$|x| = 10$

$x = \pm 10$

$\log|x| = -3$

$\log|x| = \log 10^{-3}$

$x = \pm 10^{-3}$

8. $\log_x 2 \cdot \log_{2x} 2 - \log_{4x} 2 \cdot \log_{8x} 2 = 0$

9. $5^{2(\log_5 2 + x)} - 2 = 5^{x + \log_5 2}$

8. Uslov $x>0,\ x\neq 1$

$$\log_x 2 = \frac{\log_2 2}{\log_2 x} = \frac{1}{\log_2 x}$$

$$\log_{2x} 2 = \frac{\log_2 2}{\log_2 2x} = \frac{1}{\log_2 2 + \log_2 x} = \frac{1}{1+\log_2 x}$$

$$\log_{4x} 2 = \frac{\log_2 2}{\log_2 4x} = \frac{1}{\log_2 4 + \log_2 x} = \frac{1}{2+\log_2 x}$$

$$\log_{8x} 2 = \frac{\log_2 2}{\log_2 8x} = \frac{1}{3+\log_2 x}$$

$$\frac{1}{\log_2 x}\cdot\frac{1}{1+\log_2 x} - \frac{1}{2+\log_2 x}\cdot\frac{1}{3+\log_2 x} = 0$$

$$\log_2 x = t$$

$$\frac{1}{t}\cdot\frac{1}{1+t} - \frac{1}{2+t}\cdot\frac{1}{3+t} = 0 \quad \Big/ \cdot\ t(1+t)(2+t)(3+t)\neq 0$$

$$(2+t)(3+t) - t(1+t) = 0$$

$$6+2t+3t+\cancel{t^2}-t-\cancel{t^2}=0$$

$$+4t=-6$$

$$t=-\frac{6}{4}$$

$$t=-\frac{3}{2}$$

$$\log_2 x = -\frac{3}{2}$$

$$\log_2 x = \log_2 2^{-\frac{3}{2}}$$

$$x=\frac{1}{2^{\frac{3}{2}}}$$

$$x=\frac{1}{\sqrt{8}}$$

$$x=\frac{1}{2\sqrt{2}}$$

9. $5^{2(\log_5 2 + x)} - 2 = 5^{x+\log_5 2}$

$5^{2(\log_5 2 + x)} - 5^{\log_5 2} = 5^{x+\log_5 2}$

$5^{2\log_5 2 + 2x} - 5^{\log_5 2} - 5^{x+\log_5 2} = 0$

$5^{\log_5 2 + \log_5 2 + 2x} - 5^{\log_5 2} - 5^{x+\log_5 2} = 0$

$5^{\log_5 2}\left(5^{\log_5 2 + 2x} - 1 - 5^{x}\right) = 0$

$2 \cdot \left(2 \cdot 5^{2x} - 1 - 5^{x}\right) = 0$

$2 \cdot 5^{2x} - 5^{x} - 1 = 0$

$5^{x} = t$

$2 \cdot t^{2} - t - 1 = 0$

$t_1 = -\frac{1}{2} \quad t_2 = 1$

$5^{x} = -\frac{1}{2} \quad 5^{x} = 1$

$5^{x} = 5^{0}$

$x = 0$

LOGARITAMSKE NEJEDNAČINE

1. Riješiti logaritamsku nejednačinu $\log_{0,5}(x^2-4x+3) \geq -3$.

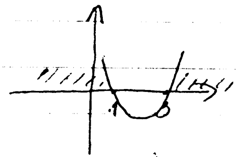
Rj. $\log_{0,5}(x^2-4x+3) \geq -3$

Definiciono područje nejednačine : $x^2-4x+3>0$

$x^2-3x-x+3>0$

$x(x-3)-(x-3)>0$

$(x-3)(x-1)>0$



$x \in (-\infty, 1) \cup (3, +\infty)$

$\log_{0,5}(x^2-4x+3) \geq \log_{0,5} 0,5^{-3}$

$x^2-4x+3 \leq 8$

$x^2-4x+3-8 \leq 0$

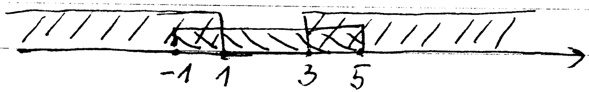
$x^2-4x-5 \leq 0$

$x^2-5x+x-5 \leq 0$

$x(x-5)+(x-5) \leq 0$

$(x-5)(x+1) \leq 0$

$x \in [-1, 5]$



Rj. $x \in [-1, 1) \cup (3, 5]$

(2) $\log_5 x \geq \log_{25}(3x-2)$

Rj. $\log_5 x \geq \log_{25}(3x-2)$

D.R. $x>0 \wedge 3x-2>0 \Rightarrow x>\frac{2}{3}$

$$\log_5 x \geq \frac{\log_5(3x-2)}{\log_5 25}$$

$$\log_5 x \geq \frac{\log_5(3x-2)}{2}$$

$$2\log_5 x \geq \log_5(3x-2)$$

$$\log_5 x^2 \geq \log_5(3x-2)$$

$$x^2 \geq 3x-2$$

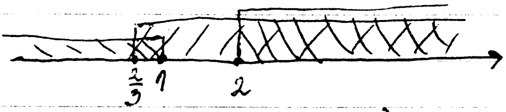
$$x^2-3x+2 \geq 0$$

$$x^2-2x-x+2 \geq 0$$

$$x(x-2)-(x-2) \geq 0$$

$$(x-2)(x-1) \geq 0$$

$$x \in (-\infty, 1] \cup [2, +\infty)$$



Rj. $x \in (\frac{2}{3}, 1] \cup [2, +\infty)$

③ $2\log_x 3 \, \log_{3x} 3 < \log_{9\sqrt{x}} 3$

D.P. $x>0 \wedge x\neq 1$

$$\log_{3x}3 = \frac{\log_3 3}{\log_3 3x} = \frac{1}{\log_3 3x} = \frac{1}{1+\log_3 x}$$

$$\log_x 3 = \frac{1}{\log_3 x}$$

$$\log_{9\sqrt{x}}3 = \frac{\log_3 3}{\log_3 9\sqrt{x}} = \frac{1}{2+\frac{1}{2}\log_3 x} = \frac{2}{4+\log_3 x}$$

$$\frac{2}{\log_3 x}\cdot\frac{1}{1+\log_3 x} < \frac{2}{4+\log_3 x} \quad \Big| \cdot\frac{1}{2}$$

$$\frac{1}{\log_3 x(1+\log_3 x)} < \frac{1}{4+\log_3 x}$$

$$\log_3 x = t$$

$$\frac{1}{t(1+t)} < \frac{1}{4+t}$$

$$\frac{1}{t(t+1)} - \frac{1}{t+4} < 0$$

$$\frac{t+4-t^2-t}{t(t+1)(t+4)} < 0$$

$$\frac{-t^2+4}{t(t+1)(t+4)} < 0$$

$$\frac{(t-2)(t+2)}{t(t+1)(t+4)} > 0$$

	$-\infty$ … -4	-4 … -2	-2 … -1	-1 … 0	0 … 2	2 … $+\infty$
$t-2$	−	−	−	−	− 0	+
$t+2$	−	− 0	+	+	+	+
t	−	−	−	− 0	+	+
$t+1$	−	−	− 0	+	+	+
$t-4$	− 0	+	+	+	+	+
	−	+	−	+	−	+

$$t\in(-4,-2)\cup(-1,0)\cup(2,+\infty)$$

$\log_3 x = -4$

$\log_3 x = \log_3 3^{-4}$

$x=\frac{1}{81}$

$\log_3 x = -2$

$x=\frac{1}{9}$

$\log_3 x = -1$

$x=\frac{1}{3}$

$\log_3 x = 0$

$x=1$

$\log_3 x = 2$

$x=9$

$$x\in\left(\frac{1}{81},\frac{1}{9}\right)\cup\left(\frac{1}{3},1\right)\cup(9,+\infty)$$

4.) $\log_3(3^x-1)+1 < \log_3(3^x+15)-x$

D.P. $3^x-1>0$

$3^x>1$

$3^x>3^0$

$x>0$

$\log_3(3^x-1)+\log_3 3^1 < \log_3(3^x+15)-\log_3 3^x$

$\log_3(3^x-1)\cdot 3 < \log_3 \frac{3^x+15}{3^x}$

$3(3^x-1) < \frac{3^x+15}{3^x} \quad \Big| \cdot 3^x \qquad 3^x = t > 1$

$3\cdot 3^{2x}-3\cdot 3^x < 3^x+15$

$3\cdot 3^{2x}-4\cdot 3^x-15<0$

$3\cdot t^2-4\cdot t-15<0$

$3(t-3)(t+\frac{5}{3})<0$

$t\in(-\frac{5}{3},3)$

$t\in(1,3)$

$3^x\in(1,3)$

$x\in(0,1)$

5. $\log_{x-3}(x^2-4x+3)<0$

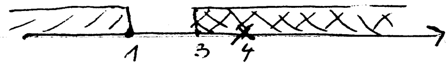
D.P. $x^2-4x+3>0 \quad \wedge \quad x-3>0 \quad \wedge \quad x-3\neq 1$

$x^2-3x-x+3>0 \qquad x>3 \qquad x\neq 4$

$x(x-3)-(x-3)>0$

$(x-3)(x-1)>0$

$x\in(-\infty,1)\cup(3,+\infty)$



D.P. $x\in(3,4)\cup(4,+\infty)$

1° $0<a<1$

$x-3>0 \Rightarrow x>3$

$x-3<1 \Rightarrow x<4$

$\Rightarrow x\in(3,4)$

$\log_{x-3}(x^2-4x+3)<\log_{x-3}(x-3)^0$

$x^2-4x+3>1$

$x^2-4x+3-1>0$

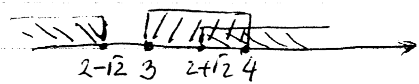
$x^2-4x+2>0$

$x_{1,2}=\frac{4\pm\sqrt{16-8}}{2}$

$x_{1,2}=\frac{4\pm2\sqrt2}{2}=2\pm\sqrt2$

$(x-(2+\sqrt2))(x-(2-\sqrt2))>0$

$x\in(-\infty,2-\sqrt2)\cup(2+\sqrt2,+\infty)$



$x\in(2+\sqrt2,4)$

2° $a>1$

$x-3>1$

$x>4$

$\log_{x-3}(x^2-4x+3)<\log_{x-3}(x-3)^0$

$x^2-4x+3<1$

$x^2-4x+3-1<0$

$x\in(2-\sqrt2,2+\sqrt2)$

~~$x\in\{\emptyset\}$~~ $x\in\{\}$ $x\in\emptyset$

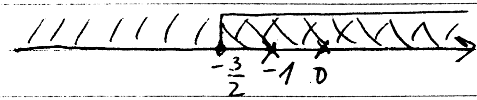
konačno rješenje $x\in(2+\sqrt2,4)$

⑥ $\log_{2x+3} x^2 < 1$

D.P. $\forall x \in \mathbb{R}\setminus\{0\} \wedge 2x+3>0 \wedge 2x+3 \neq 1$

$\wedge \quad x > -\frac{3}{2} \quad \wedge \quad 2x \neq -2$

$x \neq -1$



$x \in \left(-\frac{3}{2}, -1\right) \cup (-1, 0) \cup (0, +\infty)$

$1^\circ \quad 0 < a < 1$

$2x+3>0 \quad x > -\frac{3}{2}$

$2x+3<0 \quad x<-1$

$x \in \left(-\frac{3}{2}, -1\right)$

$\log_{2x+3} x^2 < \log_{2x+3} (2x+3)^1$

$x^2 > 2x+3$

$x^2-2x-3>0$

$x^2-3x+x-3>0$

$x(x-3)+x-3>0$

$(x-3)(x+1)>0$

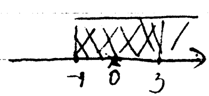
$x \in (-\infty, -1) \cup (3, +\infty)$ Rj. 1° $x \in \left(-\frac{3}{2}, -1\right)$

$2^\circ \quad a>1 \quad 2x+3>1$

$x>-1$

$(x-3)(x+1)<0$

$x \in (-1, 3)$

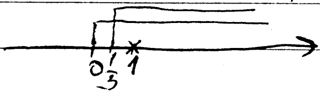


Rj. 2° $x \in (-1, 0) \cup (0, 3)$

Rj. $x \in \left(-\frac{3}{2}, -1\right) \cup (-1, 0) \cup (0, 3)$

7. $\log_x \frac{3x-1}{x^2+1} > 0$

D.P. $x>0 \wedge x\neq 1 \wedge 3x-1>0 \Rightarrow x>\frac{1}{3}$



$x\in\left(\frac{1}{3},1\right)\cup(1,+\infty)$

1° $0<x<1$

$$\log_x \frac{3x-1}{x^2+1} > \log_x x^0$$

$$\frac{3x-1}{x^2+1} < 1$$

$$\frac{3x-1-x^2-1}{x^2+1} < 0$$

$$\frac{x^2-3x+2}{x^2+1} > 0$$

$$\frac{x^2-2x-x+2}{x^2+1} > 0$$

$$\frac{x(x-2)-(x-2)}{x^2+1} > 0$$

$$\frac{(x-1)(x-2)}{x^2+1} > 0$$

$$(x-1)(x-2)>0$$

$x\in(-\infty,1)\cup(2,+\infty) \Rightarrow$ zbog D.P. $x\in\left(\frac{1}{3};1\right)$

2° $x>1$

$x\in(1,2) \Rightarrow$ rj. $x\in\left(\frac{1}{3},1\right)\cup(1,2)$

TRIGONOMETRIJSKE JEDNAČINE

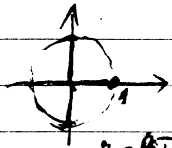
1. Riješiti trigonometrijsku jednačinu: $\sin 2x - \cos x = 0$

Rj. $\sin 2x - \cos x = 0$

$2\sin x \cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

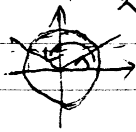
$\cos x = 0 \quad \lor \quad 2\sin x - 1 = 0$



$x_k = \frac{\pi}{2} + k\pi$

$k \in \mathbb{Z}$

$\sin x = \frac{1}{2}$



$x_n = \frac{\pi}{6} + 2n\pi \qquad n \in \mathbb{Z}$

$x_n = \pi - \frac{\pi}{6} + 2n\pi$

$x_n = \frac{5\pi}{6} + 2n\pi \qquad n \in \mathbb{Z}$

$\sin x = a, \quad -1 \le a \le 1$

$-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$

$\sin x = \sin \alpha$

$x_m = \alpha + 2m\pi$

$x_n = (\pi - \alpha) + 2n\pi$

$m, n \in \mathbb{Z}$

$\cos x = a \quad -1 \le a \le 1$

$\cos x = \cos \alpha$

$x_m = \alpha + 2m\pi$

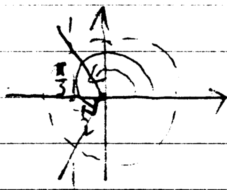
$x_n = -\alpha + 2n\pi$

$m, n \in \mathbb{Z}$

2. $\sin x + \sin 3x + \sin 5x = 0$

$\sin 3x + \sin x + \sin 5x = 0$

$\sin 3x + 2\sin \frac{x+5x}{2} \cdot \cos \frac{x-5x}{2} = 0$



$\sin 3x + 2\sin 3x \cdot \cos 2x = 0$

$\sin 3x(1 + 2\cos 2x) = 0$

$\sin 3x = 0$

$3x = t$

$\sin t = 0$

$x = k\pi$

$3x_k = k\pi$

$x_k = \frac{k\pi}{3}$

$k \in \mathbb{Z}$

$\cos 2x = -\frac{1}{2}$

$2x = t$

$\cos t = -\frac{1}{2}$

$t_1 = \pi + \frac{\pi}{3}$

$t_2 = \pi + \frac{\pi}{3}$

$2x = \frac{2\pi}{3} + 2n\pi$

$x = \frac{\pi}{3} + n\pi \quad n \in \mathbb{Z}$

$2x = \frac{4\pi}{3} + 2n\pi$

$x_n = \frac{2\pi}{3} + n\pi \quad n \in \mathbb{Z}$

3. $\cos\left(x+\frac{\pi}{6}\right)=\sin\left(x-\frac{\pi}{3}\right)$

$\cos x\cdot\cos\frac{\pi}{6}-\sin x\cdot\sin\frac{\pi}{6}=\sin x\cdot\cos\frac{\pi}{3}-\sin\frac{\pi}{3}\cos x$

$\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x=\frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\cos x$

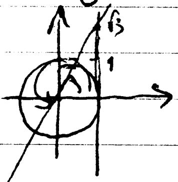
$\frac{\sqrt{3}}{2}\cos x+\frac{\sqrt{3}}{2}\cos x=\frac{1}{2}\sin x+\frac{1}{2}\sin x$

$\sqrt{3}\cos x = \sin x$

$\sin x=\sqrt{3}\cos x \quad |:\cos x\neq 0 \quad z.\ x\neq\frac{\pi}{2}+k\pi$

$\frac{\sin x}{\cos x}=\sqrt{3}$

$\operatorname{tg} x=\sqrt{3}$



$x=\frac{\pi}{3}+k\pi$

$k\in\mathbb{Z}$

4. $\sin\frac{x}{2} + \cos x = 1$

$\sin\frac{x}{2} + \cos x - 1 = 0$

$\sin\frac{x}{2} - (1 - \cos x) = 0$

Kako je

$\sin^2\frac{x}{2} = \frac{1-\cos x}{2} \Rightarrow 1 - \cos x = 2\sin^2\frac{x}{2}$

$\sin\frac{x}{2} - 2\sin^2\frac{x}{2} = 0$

$\sin\frac{x}{2}\left(1 - 2\sin\frac{x}{2}\right) = 0$

$\sin\frac{x}{2} = 0$

$\frac{x}{2} = k\pi$

$x_k = 2k\pi$

$k \in \mathbb{Z}$

$1 - 2\sin\frac{x}{2} = 0$

$2\sin\frac{x}{2} = 1$

$\sin\frac{x}{2} = \frac{1}{2}$

$\frac{x_n}{2} = \frac{\pi}{6} + 2k\pi$

$x_n = \frac{\pi}{3} + 4n\pi$

$\frac{x_n}{2} = \pi - \frac{\pi}{6} + 2n\pi$

$\frac{x_n}{2} = \frac{5\pi}{6} + 2n\pi$

$x_n = \frac{5\pi}{3} + 4n\pi \quad n \in \mathbb{Z}$

5. $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$

$\sin x + \sin 4x + \sin 2x + \sin 3x = 0$

$2\sin\frac{5x}{2}\cdot\cos\frac{3x}{2} + 2\sin\frac{5x}{2}\cos\frac{x}{2} = 0$

$2\sin\frac{5x}{2}\left(\cos\frac{3x}{2} + \cos\frac{x}{2}\right) = 0$

$2\sin\frac{5x}{2}\cdot\left(2\cos\cancel{2x}\cdot\cos\frac{x}{2}\right) = 0$

$\sin\frac{5x}{2}\cos 2x\cdot\cos x = 0$

gr. $\cos\frac{\frac{3x}{2}+\frac{x}{2}}{2} = \cos\frac{\frac{4x}{2}}{2} = \cos\frac{4x}{4} = \cos x$

$\cos\frac{\frac{3x}{2}-\frac{x}{2}}{2} = \cos\frac{\frac{2x}{2}}{2} = \cos\frac{x}{2}$

$\sin\frac{5x}{2} = 0$

$\frac{5x}{2} = k\pi$

$5x = 2k\pi$

$x_k = \frac{2k\pi}{5}$

$\cos\cancel{x} = 0$

$\cancel{2x_n = \frac{\pi}{2} + k\pi}$

$\cancel{x_n = \pi}$

$\cos\frac{x}{2} = 0$

$\frac{x}{2} = \frac{\pi}{2} + n\pi \quad /\cdot 2$

$x = \pi + 2n\pi$

$n \in \mathbb{Z}$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

Рјешавање триг. једначина $\sin x = a$, $\cos x = a$

6. $\cos 4x = -2\cos^2 x$

$\cos 2\cdot 2x = -2\cos^2 x$

$\cos^2 2x - \sin^2 2x = -2\cos^2 x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = \cos^2 x - 1 + \cos^2 x$

$\cos 2x = 2\cos^2 x - 1$

$\cos^2 x = \frac{\cos 2x + 1}{2}$

$\cos^2 2x - (1 - \cos^2 2x) = -2\cdot\frac{\cos 2x + 1}{2}$

$\cos^2 2x - \cancel{1} + \cos^2 2x = -\cos 2x - \cancel{1}$

$2\cos^2 2x + \cos 2x = 0$

$\cos 2x(2\cos 2x + 1) = 0$

$\cos 2x = 0$

$2x = \frac{\pi}{2} + k\pi$

$x_k = \frac{\pi}{4} + \frac{k\pi}{2}$

$k \in Z$

$\cos 2x = -\frac{1}{2}$

$2x = \pi - \frac{\pi}{3} + 2n\pi$

$2x = \frac{2\pi}{3} + 2n\pi \quad / \cdot\frac{1}{2}$

$x_n = \pm\frac{\pi}{3} + n\pi$

1. Naći sve realne vrijednosti za koje je definiran logaritam

$$\log_{(x^2-x-6)}(x^2+x-6)$$ (VENE)

2. Riješi logaritamske nejednačine:

a) $\log_{(2x^2-x)}(2x+2)<1$ (VENE)

b) $\log_x \frac{4x+5}{6-5x} < -1$ (VENE)

c) $\log_{x+1}(x^2+x-6)^2 \geq 4$ (MATEMATISKOP 4)

d) $\log_{(x^2-2x+1)}(3-x) \leq 1$ (-"-)

3. Riješi eksponencijalne nejednačine

a) $(x-2)^{x^2-6x+8} > 1$ (Hazim Karahmet)

b) $|x|^{x^2-x-2} < 1$ -"-

1. Riješiti trigonometrijske jednačine:

a) $5\sin^2 x + 3\sin x\cos x - 5\cos^2 x = 2$
(napisati 2 kao $2(\sin^2 x + \cos^2 x)$)

b) $\sin 2x + \cos 2x = 1 + \sqrt{6}\sin x$

c) $3\sin x - \sqrt{3}\cos x = -3$

d) $\sqrt{2}(1+\cos x) = \operatorname{ctg}\frac{x}{2}$

e) $8\sin^4 x + 13\cos 2x = 7$

f) $\sin x \sin 2x \sin 3x = \frac{1}{4}\sin 4x$

g) $\sin^2 x + \cos^2 2x + \sin^2 3x = \frac{3}{2}$

h) $\sin x + \sin 2x = \operatorname{tg} x$

i) $\sin^4 x - \cos^4 x = \frac{1}{2}$

j) $\cos x + \sin x = \frac{\cos 2x}{1-\sin 2x}$

k) $\sin 6x + 2 = 2\cos 4x$