

# KVADRATNA FUNKCIJA

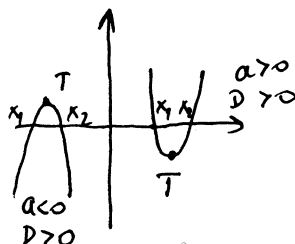
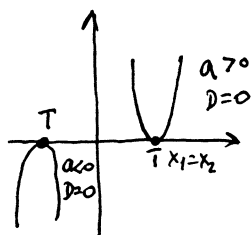
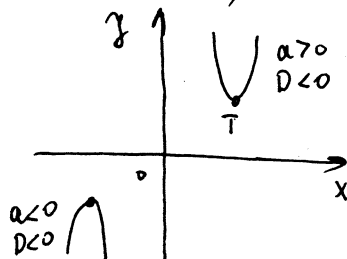
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$$f: \mathbb{R} \rightarrow \mathbb{R}$$

$$f(x) = ax^2 + bx + c, a \neq 0, a, b, c \in \mathbb{R}$$

$$f(x) = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a} \text{ kanonski oblik}$$

$$T\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$$



zadaci:

1. Za koje vrijednosti parametra  $m$  je funkcija

a)  $y = x^2 + 2(m+2)x + m+5$  neposredno za sve  $x$

b)  $y = (m+1)x^2 - 2(m+3)x + m-1$  pozitivno za sve  $x$

2. Dala je funkcija  $f(x) = (m-2)x^2 - 2mx + 2m-3$

a) za koje vrijednosti parametra  $m$  je  $f(x) < 0$  za sve  $x$ ?

b) odredi  $m$  tako da zbir recipročnih vrijednosti kvadrata korijena jednadžbe  $f(x) = 0$  bude 2.

3. Za koje vrijednosti realnog parametra  $m$  je nejednakost

$$\left| \frac{x^2 - mx + 1}{x^2 + x + 1} \right| < 3 \text{ ispunjena za sve } x?$$

4. Za koje vrijednosti parametra  $m$  je jednačina  $(m-2)x^2 - (m+1)x + (m+1) = 0$  ( $m \neq 2$ ) ima rješenja suprotnih znakova?

5. Za koje vrijednosti parametra  $m$  su korijeni jednadžbe  $(m-2)x^2 - 2mx + m-3 = 0$  ( $m \neq 2$ ) pozitivni?

## ZADACA Br. 3

I Dala je kvadratna jednačina  $(x-m)^2 = x+1$ . Odredi realne brojeve  $m$  za koje od sljedećih uslova:

a) rješenja jednačine su suprotni brojevi

b) jedna rješenje jednačine jednako je nuli

c) rješenja jednačine su realni brojevi

II Za koje vrijednosti realnog parametra  $k$  je jednačina  $(k-2)x^2 - 2kx + 3(k-3) = 0$ ,  $k \neq 2$

a) pozitivno za sve  $x$  b) za koje vrijednosti  $k$  vrijedi  $x_1 - x_2 = 1$

III Dala je kvadratna jednačina po  $x$ :  $2x^2 + 2x + c \cos \alpha = 0$  ( $0 < \alpha < \pi$ )

a) odrediti  $\alpha$  ako je  $\frac{1}{x_1} + \frac{1}{x_2} = \frac{4}{13}$

b) pokazati da je tada  $x_1^2 + x_2^2 < 1,9$ , gdje su  $x_1$  i  $x_2$  rješenja date jednačine

Решения:

(2)

① а) и б) решите сами

а) Р:  $m < -\frac{49}{9}$  г.  $m \in (-\infty, -\frac{49}{9})$

б) не имеет решения

② а)  $f(x) < 0 \Leftrightarrow a < 0 \wedge D < 0$

$$m-2 < 0 \quad \wedge \quad D = 4m^2 - 4(m-2)(2m-3) < 0$$

$$m < 2 \quad \wedge \quad D = 4m^2 - 8m^2 + 28m - 24 < 0$$

$$-4m^2 + 28m - 24 < 0 \quad / : (-4)$$

$$m^2 - 7m + 6 > 0$$

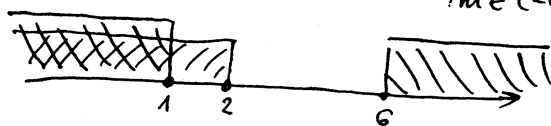
$$m^2 - 6m - m + 6 > 0$$

$$m(m-6) - (m-6) > 0$$

$$(m-1)(m-6) > 0$$

$$m \in (-\infty, 1) \cup (6, +\infty)$$

$$m < 2 \quad \wedge$$



Решение:  $m \in (-\infty, 1)$

б)  $x_1 + x_2 = \frac{2m}{m-2}$

$$x_1 \cdot x_2 = \frac{2m-3}{m-2}$$

$$\frac{1}{x_1^2} + \frac{1}{x_2^2} = 2$$

$$\frac{x_2^2 + x_1^2}{(x_1 x_2)^2} = 2$$

$$\frac{x_1^2 + x_2^2}{(x_1 x_2)^2} = 2$$

$$\frac{(x_1 + x_2)^2 - 2x_1 x_2}{(x_1 x_2)^2} = 2$$

$$\frac{4m^2 - 2(2m-3)(m-2)}{(m-2)^2} = 2$$

$$4m^2 - 4m^2 + 14m - 12 = 2(4m^2 - 12m + 9)$$

$$4m^2 - 19m + 15 = 0$$

$$m_1 = \frac{15}{4}$$

$$m_2 = 1$$

Методика обработки результатов

а)  $f(x) < 0 \Leftrightarrow a < 0 \wedge D < 0$

2. Решение неравенства  $a < 0$  и

3. Решение (проверка интервала по б.о.и)

б) 1. Вектор формулы  $x_1 + x_2 = -\frac{b}{a}$   
 $x_1 - x_2 = \frac{c}{a}$

2. Постановка задачи  $\frac{1}{x^2} + \frac{1}{x^2} = 2$

3. Доведение до многочлена на  
облик  $ax^2 + bx + c = 0, a \neq 0,$   
 $a, b, c \in \mathbb{R}$

4. Решение кв. уравн. д.е.  
решение задачи

③  $-3 < \frac{x^2 - mx + 1}{x^2 + x + 1} < 3$

$$-3 < \frac{x^2 - mx + 1}{x^2 + x + 1} \quad \wedge \quad \frac{x^2 - mx + 1}{x^2 + x + 1} < 3$$

$$\frac{x^2 - mx + 1}{x^2 + x + 1} + 3 > 0 \quad \wedge \quad \frac{x^2 - mx + 1}{x^2 + x + 1} - 3 < 0$$

$$\frac{4x^2 - (m-3)x + 4}{x^2 + x + 1} > 0 \quad \wedge \quad \frac{-2x^2 - (m+3)x - 2}{x^2 + x + 1} < 0$$

Квадратный трином

$x^2 + x + 1 > 0$  всегда, по  
формуле дискриминанта  
б.о.и

$$4x^2 - (m-3)x + 4 > 0 \quad \wedge \quad -2x^2 - (m+3)x - 2 < 0$$

$$4x^2 - (m-3)x + 4 > 0 \quad \wedge \quad 2x^2 + (m+3)x + 2 > 0$$

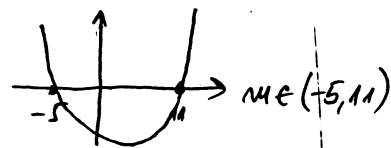
$$f(x) > 0 \Leftrightarrow a > 0 \wedge D < 0$$

$$D = (m-3)^2 - 4 \cdot 4 \cdot 4 < 0$$

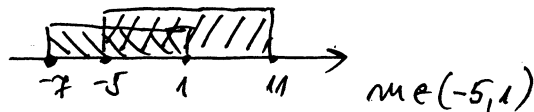
$$m^2 - 6m + 9 - 64 < 0$$

$$m^2 - 6m - 55 < 0$$

$$(m+5)(m-11) < 0$$



Rešenje :

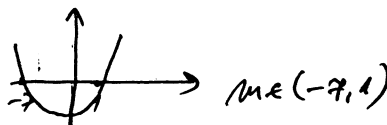


$$D = (m+3)^2 - 4 \cdot 2 \cdot 2 < 0$$

$$m^2 + 6m + 9 - 16 < 0$$

$$m^2 + 6m - 7 < 0$$

$$(m-1)(m+7) < 0$$



Metodička vred. : 6

1.  $|a| < b \Leftrightarrow -b < a < b$
2.  $f(x) > 0 \Leftrightarrow a > 0 \wedge D < 0$
3. Rešavajući nejednačinu  $a > 0$  i do grafika
4. Rešavajući zadatka (preko rešenja nejednačina na brojevnoj osi)

4)  $x_1 \cdot x_2 < 0$

$$x_1 \cdot x_2 = \frac{m+1}{m-2} < 0$$

	$-\infty$	$-1$	$2$	$+\infty$
$\frac{m+1}{m-2}$	$-$	$+$	$+$	$+$
$\frac{m-2}{m-2}$	$-$	$-$	$+$	$+$
$\frac{m+1}{m-2}$	$+$	$-$	$+$	$+$

Rešenje:  $m \in (-1, 2)$

5)  $D \geq 0 \wedge x_1 + x_2 > 0 \wedge x_1 \cdot x_2 > 0$

$$D = 4m^2 - 4(m-2)(m-3)$$

$$D = 4m^2 - 4(m^2 - 5m + 6)$$

$$D = 4m^2 - 4m^2 + 20m - 24$$

$$20m - 24 \geq 0$$

$$20m \geq 24$$

$$m \geq \frac{6}{5}$$

$$x_1 + x_2 = \frac{2m}{m-2} > 0$$

$$x_1 \cdot x_2 = \frac{m-3}{m-2} > 0$$

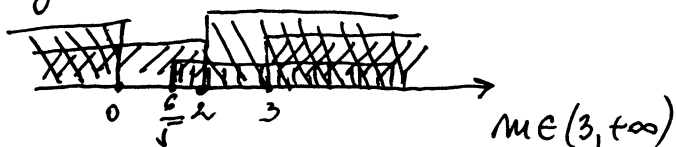
	$-\infty$	$0$	$2$	$+\infty$
$\frac{2m}{m-2}$	$-$	$+$	$+$	$+$
$\frac{m-2}{m-2}$	$-$	$-$	$+$	$+$
$\frac{2m}{m-2}$	$+$	$-$	$+$	$+$

	$-\infty$	$2$	$3$	$+\infty$
$\frac{m-3}{m-2}$	$-$	$-$	$+$	$+$
$\frac{m-2}{m-2}$	$-$	$+$	$+$	$+$
$\frac{m-3}{m-2}$	$+$	$-$	$+$	$+$

$$m \in (-\infty, 0) \cup (2, +\infty)$$

$$m \in (-\infty, 2) \cup (3, +\infty)$$

Rešenje: zad:



Metodička vred. : 6

1. Utvrditi da jednačina ima rešenje.  $D \geq 0 \wedge x_1 + x_2 > 0 \wedge x_1 \cdot x_2 > 0$  4b
2. Rešavajući dobijenu jednačinu tabelarno 3b
3. Preko rešenja nejednačina na brojevnoj osi 1b
4. Rešavajući zadatka 2b

1. Riješi eksponencijalnu jednačinu  $15 \cdot 2^{x+1} + 15 \cdot 2^{2-x} = 135$

Ry.

$$15 \cdot 2^x \cdot 2 + 15 \cdot 2^2 \cdot 2^{-x} = 135 \quad | :15$$

$$2 \cdot 2^x + 4 \cdot 2^{-x} = 9 \quad | \cdot 2^x$$

$$2 \cdot 2^{2x} + 4 - 9 \cdot 2^x = 0$$

$$2 \cdot 2^{2x} - 9 \cdot 2^x + 4 = 0$$

$$2^x = t$$

$$2 \cdot t^2 - 9t + 4 = 0$$

$$t_1 = \frac{1}{2} \quad t_2 = 4$$

$$2^x = \frac{1}{2} \quad 2^x = 4$$

$$2^x = 2^{-1} \quad 2^x = 2^2$$

$$x = -1 \quad x = 2$$

Metodička brdo i bodovanje (56)

1. Operacije sa stepenima  $a^{u \pm v} = a^u \cdot a^{\pm v}$  10

2. Rješavanje eksponencijalne jed. na kvadratnu jednačinu 20

3. Rješavanje kvadratne jed. 10

4. Rješavanje eksponencijalne jednačine  $a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x)$  10  
 $0 < a \neq 1$

2. Riješi eksponencijalnu jednačinu  $2^{|x+2|} - |2^{x+1} - 1| = 2^{x+1} + 1$

$$|x+2| = \begin{cases} x+2, & x+2 \geq 0, x \geq -2 \\ -(x+2), & x < -2 \end{cases}$$

$$|2^{x+1} - 1| = \begin{cases} 2^{x+1} - 1, & 2^{x+1} - 1 \geq 0, 2^{x+1} \geq 1, x \geq -1 \\ 1 - 2^{x+1}, & x < -1 \end{cases}$$

	$-\infty$	$-2$	$-1$	$+\infty$
$x+2$		$-$	$+$	$+$
$2^{x+1} - 1$		$-$	$-$	$+$

1°  $x \in (-\infty, -2)$

2°  $x \in [-2, -1)$

3°  $x \in [-1, +\infty)$

$$2^{-x-2} - (1 - 2^{x+1}) = 2^{x+1} + 1$$

$$2^{-x-2} - 1 + 2^{x+1} = 2^{x+1} + 1$$

$$2^{-x-2} = 2$$

$$-x-2 = 1$$

$$-x = 3$$

$$x = -3 \in (-\infty, -2)$$

$$2^{x+2} - (1 - 2^{x+1}) = 2^{x+1} + 1$$

$$2^{x+2} - 1 + 2^{x+1} = 2^{x+1} + 1$$

$$2^{x+2} = 2$$

$$x+2 = 1$$

$$x = -3 \notin [-2, -1)$$

$$2^{x+2} - 2^{x+1} + 1 = 2^{x+1} + 1$$

$$2^{x+2} - 2 \cdot 2^{x+1} = 0$$

$$2^{x+2} - 2^{x+2} = 0$$

$$\forall x \in \mathbb{R}, x \geq -1$$

Kon. rješenje:  $x = -3$  i skup  $\{x \geq -1\}$

## Metodička obrada i bodovanje:

1.  $|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}$

4b

2. Predstavljajući intervali bodovima 2b

3. Rješavanje eksponencijalnih jednačina (razlikovanje slučajeva) 3b

4. Rješavanje zadataka 1b

3. Rješati jednačinu:  $\left(\frac{4}{9}\right)^x - \left(\frac{27}{8}\right)^{x-1} = \frac{\log 4}{\log 8}$

$$\left(\left(\frac{2}{3}\right)^2\right)^x \cdot \left(\left(\frac{3}{2}\right)^3\right)^{x-1} = \frac{\log 4}{\log 8}$$

$$\left(\frac{2}{3}\right)^{2x} \cdot \left(\frac{3}{2}\right)^{3x-3} = \frac{\log 4}{\log 8}$$

$$\left(\frac{2}{3}\right)^{2x} \cdot \left(\frac{2}{3}\right)^{3-3x} = \frac{\log 2^2}{\log 2^3}$$

$$\left(\frac{2}{3}\right)^{2x} \cdot \left(\frac{2}{3}\right)^{3-3x} = \frac{2 \log 2}{3 \log 2}$$

$$\left(\frac{2}{3}\right)^{2x+3-3x} = \frac{2}{3}$$

$$\left(\frac{2}{3}\right)^{-x+3} = \frac{2}{3}$$

$$-x+3 = 1$$

$$-x = -2$$

$$x = 2$$

## Metodička obrada i bodovanje:

1. Operacije sa stepenima:  $(a^m)^n = a^{mn}$ ,  $a^{-m} = \frac{1}{a^m}$ ,  $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$   
 $a^{m+n} = a^m \cdot a^n$

2.  $\log_b a^n = n \cdot \log_b a$

3. Rješavanje eksponencijalnih jed.  $a^{f(x)} = a^{g(x)} \Leftrightarrow f(x) = g(x)$ ,  $a > 0, a \neq 1$

4. Riješiti jednačinu:  $5^{\log x} - 3^{\log x - 1} = 3^{\log x + 1} - 5^{\log x - 1}$

$\frac{1}{x}$   $5^{\log x} + 5^{\log x - 1} = 3^{\log x + 1} + 3^{\log x - 1}$   $x > 0$   
 $5^{\log x} (1 + \frac{1}{5}) = 3^{\log x} (3 + \frac{1}{3})$

$$5^{\log x} \frac{6}{5} = 3^{\log x} \frac{10}{3}$$

$$5^{\log x} = 3^{\log x} \cdot \frac{10}{3} \cdot \frac{5}{8} = \frac{5}{3}$$

$$\frac{5^{\log x}}{3^{\log x}} = \frac{5^2}{3^2}$$

$$\left(\frac{5}{3}\right)^{\log x} = \left(\frac{5}{3}\right)^2$$

$$\log x = 2$$

$$\log x = \log 10^2$$

$\frac{1}{x}$   $x = 100$

Metodičko obrađeno i bod. (10b)

1. Grupiranje jedna jedinst. b. 2b
2. Ispitivanje ekv. jedn. 4b
3. Ispitivanje log. jedn. 3b
4. Definicijom područje l. j. 1b
5. 1

5.  $\log_x 3 - \log_x 2 = \frac{1}{2}$

$x > 0, x \neq 1$

$$\log_x 3^{\frac{1}{2}} = \frac{1}{2}$$

$$\log_x 3^{\frac{1}{2}} = \log_x x^{\frac{1}{2}}$$

$$\frac{1}{2} = \frac{1}{2}$$

$$\sqrt{x} = \frac{3}{2}$$

$\frac{1}{x}$   $x = \frac{9}{4} > 0$

Metodičko obrađeno i bod. (5b)

1.  $y = \log_a x, a > 0, a \neq 1$  1b
2.  $\log_a x - \log_a y = \log_a \frac{x}{y}$  2b
3.  $\log_a a^k = k$  1b
4. Rješivanje log. jednačine 1b

6. Riješiti log. jednačinu  $\log_2 \sqrt{1-x}^2 = 3$

$$\log_2 |1-x| = 3$$

$x-1 \neq 0, x \neq 1$

$$|1-x| = \begin{cases} 1-x, & -x \geq -1, x \leq 1 \\ x-1, & x > 1 \end{cases}$$

$1^\circ x \in (-\infty, 1)$

$$\log_2 (1-x) = 3$$

$$\log_2 (1-x) = \log_2 2^3$$

$$1-x = 8$$

$$-x = 7$$

$$x = -7 \in (-\infty, 1)$$

$2^\circ x \in (1, +\infty)$

$$\log_2 (x-1) = 3$$

$$\log_2 (x-1) = \log_2 2^3$$

$$x-1 = 8$$

$$x = 9 \in (1, +\infty)$$

## Metodički obradi: (10 b)

1.  $|a^2| = |a|^2$       2. b
2.  $|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}$       2. b
3. Rješavanje logaritamskih jednačina 3+3

7. Rješiti log. jednačinu  $\log x^2 + \log^2 |x| = 3$

R.  $2\log |x| + \log^2 |x| = 3$   
 $\log^2 |x| + 2\log |x| - 3 = 0$   
 $\log |x| = t$

Definiciono područje:  $x \neq 0$

$t^2 + 2t - 3 = 0$

$t_1 = -3, \quad t_2 = 1$

$\log |x| = 1$

$\log |x| = -3$

$\log |x| = \log 10$

$\log |x| = \log 10^{-3}$

$|x| = 10$

$x = \pm 10^{-3}$

$x = \pm 10$

## Metodički obradi:

1. Definiciono područje log. jednačine
2.  $\log a^2 = 2 \log |a|$        $(\sqrt{x})^2 = |x|^2$
3. Rješavanje log. jedn. rješeno (proštenje na kv. jedn.)
4. Rješavanje log. jedn. s dodatkom na D.P.

2.  $\log_x 2 \cdot \log_x 2 - \log_{4x} 2 \cdot \log_{8x} 2 = 0$

9.  $5^2(\log_5 2 + x) - 2 = 5^x + \log_5 2$

8. Uслов  $x > 0, x \neq 1$

$$\log_x 2 = \frac{\log_2 2}{\log_2 x} = \frac{1}{\log_2 x}$$

$$\log_{2x} 2 = \frac{\log_2 2}{\log_2 2x} = \frac{1}{\log_2 2 + \log_2 x} = \frac{1}{1 + \log_2 x}$$

$$\log_{4x} 2 = \frac{\log_2 2}{\log_2 4x} = \frac{1}{\log_2 4 + \log_2 x} = \frac{1}{2 + \log_2 x}$$

$$\log_{8x} 2 = \frac{\log_2 2}{\log_2 8x} = \frac{1}{3 + \log_2 x}$$

$$\frac{1}{\log_2 x} \cdot \frac{1}{1 + \log_2 x} - \frac{1}{2 + \log_2 x} \cdot \frac{1}{3 + \log_2 x} = 0$$

$$\log_2 x = t$$

$$\frac{1}{t} \cdot \frac{1}{1+t} - \frac{1}{2+t} \cdot \frac{1}{3+t} = 0 \quad | \cdot t(1+t)(2+t)(3+t) \neq 0$$

$$(2+t)(3+t) - t(1+t) = 0$$

$$6 + 2t + 3t + t^2 - t - t^2 = 0$$

$$4t = -6$$

$$t = -\frac{6}{4}$$

$$t = -\frac{3}{2}$$

$$\log_2 x = -\frac{3}{2}$$

$$\log_2 x = \log_2 2^{-\frac{3}{2}}$$

$$x = \frac{1}{2^{\frac{3}{2}}}$$

$$x = \frac{1}{\sqrt{8}}$$

$$x = \frac{1}{2\sqrt{2}}$$

Методическая обработка: 1.  $y = \log_a x$   $a > 1$

2. Переложить на одну лог. базу на другую  $\log_6 a = \frac{\log_x a}{\log_x 6}$   $0 < b \neq 1, x \neq 1$

3. Выяснение лог. ждн. выражений

4.



$$9. 5^{2(\log_5 2 + x)} - 2 = 5^{x + \log_5 2}$$

$$5^{2(\log_5 2 + x)} - 5^{\log_5 2} = 5^{x + \log_5 2}$$

$$5^{2\log_5 2 + 2x} - 5^{\log_5 2} - 5^{x + \log_5 2} = 0$$

$$5^{\log_5 2 + \log_5 2 + 2x} - 5^{\log_5 2} - 5^{x + \log_5 2} = 0$$

$$5^{\log_5 2} (5^{\log_5 2 + 2x} - 1 - 5^x) = 0$$

$$2. (2 \cdot 5^{2x} - 1 - 5^x) = 0$$

$$2 \cdot 5^{2x} - 5^x - 1 = 0$$

$$5^x = t$$

$$2 \cdot t^2 - t - 1 = 0$$

$$b_1 = -\frac{1}{2} \quad b_2 = 1$$

$$5^x = -\frac{1}{2} \quad 5^x = 1$$

$$5^x = 5^0$$

$$x - x = 0$$

Metodyka obrade:

1.  $a^{\log_a x} = x \quad 0 < a \neq 1, x > 0$

2. Iščuvanje ek. j. skupnosti

3. Iščuvanje zbirane

# VJEŽBE br. 5

## LOGARITAMSKE NEJEDNAČINE

① Rješiti logaritamsku nejednačinu  $\log_{0,5}(x^2-4x+3) \geq -3$ .

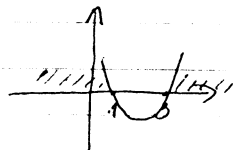
Rj.  $\log_{0,5}(x^2-4x+3) \geq -3$

Definiciono područje nejednačine :  $x^2-4x+3 > 0$

$$x^2-3x-x+3 > 0$$

$$x(x-3)-(x-3) > 0$$

$$(x-3)(x-1) > 0$$



$$x \in (-\infty, 1) \cup (3, +\infty)$$

$$\log_{0,5}(x^2-4x+3) \geq \log_{0,5} 0,5^{-3}$$

$$x^2-4x+3 \leq 8$$

$$x^2-4x+3-8 \leq 0$$

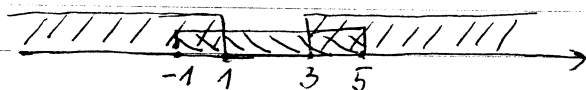
$$x^2-4x-5 \leq 0$$

$$x^2-5x+x-5 \leq 0$$

$$x(x-5)+(x-5) \leq 0$$

$$(x-5)(x+1) \leq 0$$

$$x \in [-1, 5]$$



Rj.  $x \in [-1, 1) \cup (3, 5]$

Metodički obradi : zadavanje (5b)

1. Definiciono područje logaritamske nejednačine  
(1)  $\log_a f(x) \geq k$ ,  $f(x) > 0$  ; b

$$2 < a \neq 1$$

$$(1) \Leftrightarrow \begin{cases} f(x) > 0 \\ f(x) \leq a^k \end{cases}$$

3. Rješavanje kvadratne nejednačine z.b

4. Rješavanje logaritamske nejednačine u odnosu na def. područje z.b

2)  $\log_5 x \geq \log_5 (3x-2)$

4.  $\log_5 x \geq \log_5 (3x-2)$

O.P.  $x > 0$  i  $3x-2 > 0 \Rightarrow x > \frac{2}{3}$

$$\log_5 x \geq \frac{\log_5 (3x-2)}{\log_5 25}$$

$$\log_5 x \geq \frac{\log_5 (3x-2)}{2}$$

$$2 \log_5 x \geq \log_5 (3x-2)$$

$$\log_5 x^2 \geq \log_5 (3x-2)$$

$$x^2 \geq 3x-2$$

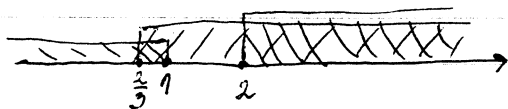
$$x^2 - 3x + 2 \geq 0$$

$$x^2 - 2x - x + 2 \geq 0$$

$$x(x-2) - (x-2) \geq 0$$

$$(x-2)(x-1) \geq 0$$

$$x \in (-\infty, 1] \cup [2, +\infty)$$



4.  $x \in [\frac{2}{3}, 1] \cup [2, +\infty)$

Metodyczne obliczenia i badanie

1.  $\log_a f(x) \geq \log_a g(x)$

Definicja: podaje  $f(x) > 0$  i  $g(x) > 0$  ( $a > 1$  i  $b > 1$ )

2. Przekształć na jedną log. base na drugą

$$\log_a u = \frac{\log_a u}{\log_a b}$$

3.  $\log_a f(x) \geq \log_a g(x)$  (1)

$$(1) \Leftrightarrow \begin{cases} f(x) > 0 \\ g(x) > 0 \\ f(x) \geq g(x) \end{cases}$$

$$(3) 2 \log_x 3 \log_{3x} 3 < \log_{9x} 3$$

$$D.P. \quad x > 0 \wedge x \neq 1$$

$$\log_{3x} 3 = \frac{\log_3 3}{\log_3 3x} = \frac{1}{\log_3 3x} = \frac{1}{1 + \log_3 x}$$

$$\log_x 3 = \frac{1}{\log_3 x}$$

$$\log_{9x} 3 = \frac{\log_3 3}{\log_3 9x} = \frac{1}{2 + \frac{1}{2} \log_3 x} = \frac{2}{4 + \log_3 x}$$

$$\frac{2}{\log_3 x} \cdot \frac{1}{1 + \log_3 x} < \frac{2}{4 + \log_3 x} \quad | \cdot \frac{1}{2}$$

$$\frac{1}{\log_3 x (1 + \log_3 x)} < \frac{1}{4 + \log_3 x}$$

$$\log_3 x = t$$

$$\frac{1}{t(1+t)} < \frac{1}{4+t}$$

$$\frac{1}{t(t+1)} - \frac{1}{t+4} < 0$$

$$\frac{t+4-t^2-t}{t(t+1)(t+4)} < 0$$

$$\frac{-t^2+4}{t(t+1)(t+4)} < 0$$

$$\frac{(t-2)(t+2)}{t(t+1)(t+4)} > 0$$

	$-\infty$	$-4$	$-2$	$-1$	$0$	$2$	$+\infty$
$t-2$	-	-	-	-	-	+	+
$t+2$	-	-	0	+	+	+	+
$t$	-	-	-	-	0	+	+
$t+1$	-	-	-	0	+	+	+
$t-4$	-	0	+	+	+	+	+
	-	(+)	-	(+)	-	(+)	

$$t \in (-4, -2) \cup (-1, 0) \cup (2, +\infty)$$

$$\log_3 x = -4$$

$$\log_3 x = \log_3 3^{-4}$$

$$x = \frac{1}{81}$$

$$\log_3 x = -2$$

$$x = \frac{1}{9}$$

$$\log_3 x = -1$$

$$x = \frac{1}{3}$$

$$\log_3 x = 0$$

$$x = 1$$

$$\log_3 x = 2$$

$$x = 9$$

$$x \in \left(\frac{1}{81}, \frac{1}{9}\right) \cup \left(\frac{1}{3}, 1\right) \cup (9, +\infty)$$

Metode drži: bočnice

$$1. y = \log_a x, \quad a > 0 \wedge a \neq 1$$

$$2. \text{Preoblikovanje na isto bazu log. baze mo dugu } \log_a a = \frac{\log_c a}{\log_c a}$$

3. Preoblikovanje na isto bazu log. baze mo dugu

4. Preoblikovanje log. na isto bazu

④.  $\log_3(3^x-1)+1 < \log_3(3^x+15)-x$

D.P.  $3^x-1 > 0$

$3^x > 1$

$3^x > 3^0$

$x > 0$

$\log_3(3^x-1) + \log_3 3^1 < \log_3(3^x+15) - \log_3 3^x$

$\log_3(3^x-1) \cdot 3 < \log_3 \frac{3^x+15}{3^x}$

$3(3^x-1) < \frac{3^x+15}{3^x} \quad | \cdot 3^x \quad 3^x = t > 1$

$3 \cdot 3^{2x} - 3 \cdot 3^x < 3^x + 15$

$3 \cdot 3^{2x} - 4 \cdot 3^x - 15 < 0$

$3 \cdot t^2 - 4t - 15 < 0$

$3(t-3)(t+\frac{5}{3}) < 0$

$t \in (-\frac{5}{3}, 3)$

$t \in (1, 3)$

$3^x \in (1, 3)$

$x \in (0, 1)$

Методичка задача 1. Реш. 10 б

1. Определить область

2 б

2. Найти логарифмическую

$k = \log_a a^t$

$\log_a b + \log_a c = \log_a bc$

$0 < a \neq 1$

3 б

$\log_a b - \log_a c = \log_a \frac{b}{c}$

$b > 0, c > 0$

3. Решить: экспоненциальное уравнение 2 б

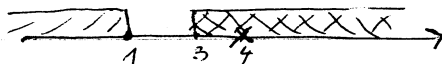
4. Решить: логарифмическое уравнение 3 б

5)  $\log_{x-3}(x^2-4x+3) < 0$

D.P.  $x^2-4x+3 > 0$   $\wedge$   $x-3 > 0$   $\wedge$   $x-3 \neq 1$   
 $x^2-3x-x+3 > 0$   $x > 3$   $x \neq 4$

$x(x-3)-(x-3) > 0$   
 $(x-3)(x-1) > 0$

$x \in (-\infty, 1) \cup (3, +\infty)$



D.P.  $x \in (3, 4) \cup (4, +\infty)$

1°  $0 < a < 1$

$x-3 > 0 \Rightarrow x > 3$

$x-3 < 1 \Rightarrow x < 4$

g.  $x \in (3, 4)$

$\log_{x-3}(x^2-4x+3) < \log_{x-3}(x-3)^0$

$x^2-4x+3 > 1$

$x^2-4x+3-1 > 0$

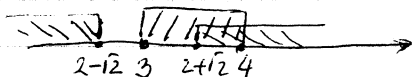
$x^2-4x+2 > 0$

$x_{1,2} = \frac{4 \pm \sqrt{16-8}}{2}$

$x_{1,2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$

$(x-(2+\sqrt{2}))(x-(2-\sqrt{2})) > 0$

$x \in (-\infty, 2-\sqrt{2}) \cup (2+\sqrt{2}, +\infty)$



$x \in (2+\sqrt{2}, 4)$

2°  $a > 1$

$x-3 > 1$

$x > 4$

$\log_{x-3}(x^2-4x+3) < \log_{x-3}(x-3)^0$

$x^2-4x+3 < 1$

$x^2-4x+3-1 < 0$

$x \in (2-\sqrt{2}, 2+\sqrt{2})$

~~$x \in (-\infty, 2-\sqrt{2}) \cup (2+\sqrt{2}, +\infty)$~~   $x \in (3, 4)$   $x \in 4$

konечно rješenje  $x \in (2+\sqrt{2}, 4)$

Metodička obrada 10.6

1.  $\log_a f(x) < k$  (1)

2.  $0 < a < 1$

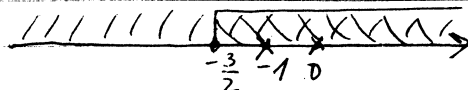
(1)  $\Leftrightarrow \begin{cases} f(x) > 0 \\ f(x) > a^k \end{cases}$  4b

3.  $a > 1$  (1)  $\Leftrightarrow \begin{cases} f(x) > 0 \\ f(x) < a^k \end{cases}$  4b

4. konačno rješenje jedu. (1) 2b

$$(6) \log_{2x+3} x^2 < 1$$

$$\begin{aligned} \text{D.P. } \forall x \in \mathbb{R} \text{ to } 1 \quad & 2x+3 > 0 \quad \wedge \quad 2x+3 \neq 1 \\ & \wedge \quad x > -\frac{3}{2} \quad \wedge \quad 2x \neq -2 \\ & & x \neq -1 \end{aligned}$$



$$x \in \left(-\frac{3}{2}, -1\right) \cup (-1, 0) \cup (0, +\infty)$$

$$1^\circ 0 < a < 1$$

$$2x+3 > 0 \quad x > -\frac{3}{2}$$

$$2x+3 < 0 \quad x < -1$$

$$x \in \left(-\frac{3}{2}, -1\right)$$

$$\log_{2x+3} x^2 < \log_{2x+3} (2x+3)'$$

$$x^2 > 2x+3$$

$$x^2 - 2x - 3 > 0$$

$$x^2 - 3x + x - 3 > 0$$

$$x(x-3) + x-3 > 0$$

$$(x-3)(x+1) > 0$$

$$x \in (-\infty, -1) \cup (3, +\infty)$$

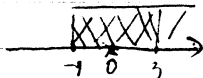
$$\text{I. } x \in \left(-\frac{3}{2}, -1\right)$$

$$2^\circ a > 1 \quad 2x+3 > 1$$

$$x > -1$$

$$(x-3)(x+1) < 0$$

$$x \in (-1, 3)$$



$$\text{I. } x \in (-1, 0) \cup (0, 3)$$

$$\text{I. } x \in \left(-\frac{3}{2}, -1\right) \cup (-1, 0) \cup (0, 3)$$

Metodicko obzrk i bod.

Let's see prelozili podatke

$$(7.) \log_x \frac{3x-1}{x^2+1} > 0$$

$$\text{d.p. } x > 0 \wedge x \neq 1 \wedge 3x-1 > 0 \Rightarrow x > \frac{1}{3}$$

$$\begin{array}{c} \text{---} | \text{---} | \text{---} \rightarrow \\ \text{0} \quad \frac{1}{3} \quad 1 \end{array} \quad x \in \left(\frac{1}{3}, 1\right) \cup (1, +\infty)$$

$$1^\circ \quad 0 < x < 1$$

$$\log_x \frac{3x-1}{x^2+1} > \log_x x^0$$

$$\frac{3x-1}{x^2+1} < 1$$

$$\frac{3x-1-x^2-1}{x^2+1} < 0$$

$$\frac{x^2-3x+2}{x^2+1} > 0$$

$$\frac{x^2-2x-x+2}{x^2+1} > 0$$

$$\frac{x(x-2)-(x-2)}{x^2+1} > 0$$

$$\frac{(x-1)(x-2)}{x^2+1} > 0$$

$$(x-1)(x-2) > 0$$

$$x \in (-\infty, 1) \cup (2, +\infty) \Rightarrow \text{269}^\circ \text{ d.p. } x \in \left(\frac{1}{3}, 1\right)$$

$$2^\circ \quad x > 1$$

$$x \in (1, 2) \Rightarrow \text{27}^\circ \quad x \in \left(\frac{1}{3}, 1\right) \cup (1, 2)$$



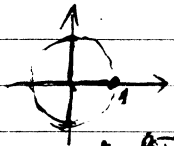
# WEZBE Br. 6

## TRIGONOMETRIJSKE JEDNAČINE

1. Riješi trigonometrijsku jednačinu:  $\sin 2x - \cos x = 0$

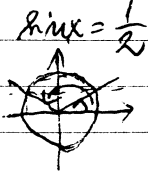
$$\begin{aligned} \sin 2x - \cos x &= 0 \\ 2\sin x \cos x - \cos x &= 0 \\ \cos x (2\sin x - 1) &= 0 \end{aligned}$$

$$\cos x = 0 \quad \vee \quad 2\sin x - 1 = 0$$



$$x_k = \frac{\pi}{2} + k\pi$$

$k \in \mathbb{Z}$



$$\sin x = \frac{1}{2}$$

$$x_m = \frac{\pi}{6} + 2m\pi \quad m \in \mathbb{Z}$$

$$x_n = \pi - \frac{\pi}{6} + 2n\pi$$

$$x_n = \frac{5\pi}{6} + 2n\pi \quad n \in \mathbb{Z}$$

### Metoda tačka i bod

1. Sinus dvostrukog ugla  $\sin 2x = 2\sin x \cos x$

2. Kosinusupitno je  $\sin x \cos x = 0$

3.  $\sin x = 0$  ili  $\cos x = 0$

$$4. \sin x = a, \quad -1 \leq a \leq 1$$

$$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$\sin x = \sin u$$

$$x_m = u + 2m\pi$$

$$x_n = (\pi - u) + 2n\pi$$

$$m, n \in \mathbb{Z}$$

$$\cos x = a, \quad -1 \leq a \leq 1$$

$$\cos x = \cos u$$

$$x_k = u + 2k\pi$$

$$x_n = -u + 2n\pi$$

$m, n \in \mathbb{Z}$

2.  $\sin x + \sin 2x + \sin 3x = 0$

$$\sin 3x + \sin x + \sin 5x = 0$$

$$\sin 3x + 2\sin \frac{x+5x}{2} \cdot \cos \frac{x-5x}{2} = 0$$

$$\sin 3x + 2\sin 3x \cdot \cos 2x = 0$$

$$\sin 3x (1 + 2\cos 2x) = 0$$

$$\sin 3x = 0 \quad \cos 2x = -\frac{1}{2}$$

$$3x = k\pi$$

$$\sin t = 0$$

$$x = k\pi$$

$$3x_k = k\pi$$

$$x_k = \frac{k\pi}{3}$$

$$k \in \mathbb{Z}$$

$$2x = t$$

$$\cos t = -\frac{1}{2}$$

$$t_1 = \pi + \frac{\pi}{3}$$

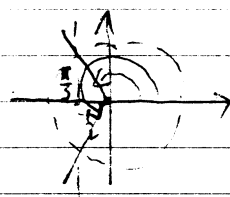
$$t_2 = \pi - \frac{\pi}{3}$$

$$2x = \frac{2\pi}{3} + 2m\pi$$

$$x = \frac{\pi}{3} + m\pi \quad m \in \mathbb{Z}$$

$$2x = \frac{4\pi}{3} + 2n\pi$$

$$x_n = \frac{2\pi}{3} + n\pi \quad n \in \mathbb{Z}$$



## Metodika obrade i bod.

1.  $\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$
  2. Kontinuitet na RRBre
  3. Rješavanje trig. jednačina  $\sin x = a$ ,  $\cos x = a$
3.  $\cos(x + \frac{\pi}{6}) = \sin(x - \frac{\pi}{3})$

$$\cos x \cos \frac{\pi}{6} - \sin x \sin \frac{\pi}{6} = \sin x \cos \frac{\pi}{3} - \sin \frac{\pi}{3} \cos x$$

$$\frac{\sqrt{3}}{2} \cos x - \frac{1}{2} \sin x = \frac{1}{2} \sin x - \frac{\sqrt{3}}{2} \cos x$$

$$\frac{\sqrt{3}}{2} \cos x + \frac{\sqrt{3}}{2} \cos x = \frac{1}{2} \sin x + \frac{1}{2} \sin x$$

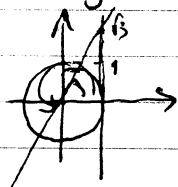
$$\sqrt{3} \cos x = \sin x$$

$$\sin x = \sqrt{3} \cos x \quad | : \cos x \neq 0 \quad \text{B. } x \neq \frac{\pi}{2} + k\pi$$

$$\frac{\sin x}{\cos x} = \sqrt{3}$$

$$\tan x = \sqrt{3}$$

$$\tan x = \sqrt{3}$$



$$x = \frac{\pi}{3} + k\pi$$
$$k \in \mathbb{Z}$$

## Metodika obrade i bod.

1. Adicione formule  $\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$   
 $\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$
2.  $\lg x = a$  tack  
 $\lg x = \lg d$   
 $x = d + k\pi \quad k = 0, 1, 2, \dots \quad k \in \mathbb{Z} \quad , d \neq \frac{\pi}{2} + k\pi$

$$4. \sin \frac{x}{2} + \cos x = 1$$

$$\sin \frac{x}{2} + \cos x - 1 = 0$$

$$\sin \frac{x}{2} - (1 - \cos x) = 0$$

Korkei

$$\sin \frac{x}{2} = \frac{1 - \cos x}{2} \Rightarrow 1 - \cos x = 2 \sin^2 \frac{x}{2}$$

$$\sin \frac{x}{2} - 2 \sin^2 \frac{x}{2} = 0$$

$$\sin \frac{x}{2} (1 - 2 \sin \frac{x}{2}) = 0$$

$$\sin \frac{x}{2} = 0$$

$$\frac{x}{2} = k\pi$$

$$x_k = 2k\pi$$

$k \in \mathbb{Z}$

$$1 - 2 \sin \frac{x}{2} = 0$$

$$2 \sin \frac{x}{2} = 1$$

$$\sin \frac{x}{2} = \frac{1}{2}$$

$$\frac{x_k}{2} = \frac{\pi}{6} + 2k\pi$$

$$x_k = \frac{\pi}{3} + 4k\pi$$

$$\frac{x_k}{2} = \pi - \frac{\pi}{6} + 2n\pi$$

$$\frac{x_k}{2} = \frac{5\pi}{6} + 2n\pi$$

$$x_k = \frac{5\pi}{3} + 4n\pi \quad n \in \mathbb{Z}$$

Metodicko obrado i bod.

$$1. \sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}$$

2. Rastavljanje na faktore

$$3. \sin x = a$$

$$5. \sin x + \sin 2x + \sin 3x + \sin 4x = 0$$

$$\sin x + \sin 4x + \sin 2x + \sin 3x = 0$$

$$2 \sin \frac{5x}{2} \cdot \cos \frac{3x}{2} + 2 \sin \frac{5x}{2} \cos \frac{x}{2} = 0$$

$$2 \sin \frac{5x}{2} (\cos \frac{3x}{2} + \cos \frac{x}{2}) = 0$$

$$2 \sin \frac{5x}{2} \cdot (2 \cos 2x \cdot \cos x) = 0$$

$$\sin \frac{5x}{2} \cos 2x \cdot \cos x = 0$$

$$\text{gr. } \cos \frac{\frac{3x}{2} + \frac{x}{2}}{2} = \cos \frac{\frac{4x}{2}}{2} = \cos \frac{2x}{2} = \cos x$$

$$\cos \frac{\frac{5x}{2} - \frac{x}{2}}{2} = \cos \frac{\frac{4x}{2}}{2} = \cos \frac{2x}{2} = \cos x$$

$$\sin \frac{5x}{2} = 0$$

$$\frac{5x}{2} = k\pi$$

$$5x = 2k\pi$$

$$x_k = \frac{2k\pi}{5}$$

$$\cos x = 0$$

$$x_k = \frac{\pi}{2} + k\pi$$

$$x_k = \frac{\pi}{2} + k\pi$$

$$\cos \frac{x}{2} = 0$$

$$\frac{x}{2} = \frac{\pi}{2} + m\pi \quad / \cdot 2$$

$$x = \pi + 2m\pi$$

$m \in \mathbb{Z}$

## Методы Бради: Бод:

$$1. \sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$2. \cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$3. \text{Граничные значения: } \sin x = a, \cos x = a$$

$$6. \cos 4x = -2 \cos^2 x$$

$$\cos 2 \cdot 2x = -2 \cos^2 x$$

$$\cos^2 2x - \sin^2 2x = -2 \cos^2 x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos 2x = \cos^2 x - 1 + \cos^2 x$$

$$\cos 2x = 2 \cos^2 x - 1$$

$$\cos^2 x = \frac{\cos 2x + 1}{2}$$

$$\cos^2 2x - (1 - \cos^2 2x) = -2 \cdot \frac{\cos 2x + 1}{2}$$

$$\cos^2 2x - 1 + \cos^2 2x = -\cos 2x - 1$$

$$2 \cos^2 2x + \cos 2x = 0$$

$$\cos^2 2x (2 \cos 2x + 1) = 0$$

$$\cos 2x = 0$$

$$\cos 2x = -\frac{1}{2}$$

$$2x = \frac{\pi}{2} + k\pi$$

$$2x = \pi - \frac{\pi}{3} + 2n\pi$$

$$x = \frac{\pi}{4} + \frac{k\pi}{2}$$

$$2x = \frac{2\pi}{3} + 2n\pi \quad | \cdot \frac{1}{2}$$

$$k \in \mathbb{Z}$$

$$x_n = \frac{\pi}{3} + n\pi$$

## Методы Бради: Бод:

$$1. \cos 2x = \cos^2 x - \sin^2 x \Rightarrow \cos^2 x = \frac{\cos 2x + 1}{2}$$

$$2. \text{Крат. на 2}$$

$$3. \cos x = a$$

## Zadaca br. 4

1. Naći sve rešenja nejednakosti za koje je definiran logaritam

$$\log(x^2 - x - 6)(x^2 + x - 6) \quad (\text{VENE})$$

2. Reši logaritamske nejednakosti:

$$a) \log(2x^2 - x)(2x + 2) < 1 \quad (\text{VENE}) \quad c) \log_{x+1}(x^2 + x - 6)^2 > 4 \quad (\text{MATEMATIKOS 4})$$

$$b) \log_x \frac{4x+5}{6x-5x} < -1 \quad (\text{VENE}) \quad d) \log(x^2 - 2x + 1)(3-x) \leq 1 \quad (-1)$$

3. Reši eksponencijalne nejednakosti

$$a) (x-2)^{x^2 - 3x + 8} > 1 \quad (\text{Hazić Kerschner})$$

$$b) |x|^{x^2 - x - 2} < 1 \quad -11$$

## Zadaca br. 5

1. Reši trigonometrijske jednačine:

$$a) 5\sin^2 x + 3\sin x \cos x - 5\cos^2 x = 2$$

(napisati 2 kao  $2(\sin^2 x + \cos^2 x)$ )

$$b) \sin 2x + \cos 2x = 1 + \sqrt{6} \sin x$$

$$c) 3\sin x - \sqrt{3} \cos x = -3$$

$$d) \sqrt{2}(1 + \cos x) = \csc \frac{x}{2}$$

$$e) 8\sin^4 x + 13\cos 2x = 7$$

$$f) \sin x \sin 2x \sin 3x = \frac{1}{4} \sin 4x$$

$$g) \sin^2 x + \cos^2 2x + \sin^2 3x = \frac{3}{2}$$

$$h) \sin x + \sin 2x = \frac{1}{\cos x}$$

$$i) \sin^4 x - \cos^4 x = \frac{1}{2}$$

$$j) \cos x + \sin x = \frac{\cos 2x}{1 - \sin 2x}$$

$$k) \sin 6x + 2 = 2\cos 4x$$

Metodički obraditi 5 jednačina.